Econ 204A: Section 4

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Announcements

- Midterm next Tuesday (25 October 2016) in class
- The material you are responsible for: essentially everything covered in class
- There are a lot of good practice problems in the HW file
- Dan's Tips:
  1. Have fun
  2. Be yourself
  3. Give it your best

Solving for the Optimal Consumption Path

- Last week we only briefly saw the set up for these types of problems
- On the HW you got a little flavor of how to work through one of them
- Today, let's actually set up and solve one of these problems. Consider the following HH problem.

$$\max U = \int_0^\infty e^{-\rho t} u(C(t)) dt \qquad \text{s.t.} \qquad \dot{a} = r(t)a(t) + W(t) - C(t)$$

$$\text{where} \qquad u(C(t)) = \frac{C(t)^{1-\theta}}{1-\theta}$$

Hamiltonian and Maximum Principle

$$\mathcal{H}(c, a, \lambda, t) = e^{-\rho t}u(C) + \lambda[ra + W - C] \tag{1}$$

$$\frac{\partial \mathcal{H}}{\partial C} = 0 : \qquad \lambda = e^{-\rho t}u'(C) \tag{2}$$

$$\frac{\partial \mathcal{H}}{\partial a} = -\dot{\lambda} : \quad \frac{\dot{\lambda}}{\lambda} = -r \tag{3}$$

$$\frac{\partial \mathcal{H}}{\partial \lambda} = \dot{a} : \qquad \dot{a} = ra + W - C \tag{4}$$

- Now we can move these results around to get the Euler Equation

Obtaining the Euler Equation

- Take logs of (2) and then take a time derivative

$$ln(\lambda) = -\rho t + ln\left(u'(C)\right)$$

$$\implies \qquad \frac{\dot{\lambda}}{\lambda} = -\rho + \frac{u''(C)}{u'(C)}\dot{C}$$

$$= -\rho + \left(\frac{u''(C)C}{u'(C)}\right)\frac{\dot{C}}{C}$$

$$= -\rho - \theta\frac{\dot{C}}{C} \tag{5}$$

Note: $-\frac{u'(C)}{u''(C)C} = \frac{1}{\theta(C)}$. For power utility, the EIS is equal to $1/\theta$. Furthermore, in order for there to be a steady state in this sort of an environment, we need θ (the measure of relative risk aversion) not to be time varying.

- Equate (3) and (5) and solve for $\dot{C}/C$

$$-r = -\rho - \theta \frac{\dot{C}}{C}$$

$$\frac{\dot{C}}{C} = \frac{r(t) - \rho}{\theta} \tag{6}$$

  Which is the EE. (6) says that per-capita consumption growth is proportional to the interest rate less the rate of time preference, adjusted by the consumer's willingness to substitute over time.

- Notice that I have been explicit about writing the interest rate as a function of time (this will come up in a bit)