Econ 204A: Section 10

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Announcements

- **Final Exam: Tuesday, 6 December 2016, 4:00p - 7:00p**
- Review session: Saturday, 3 December 2016, 11:00a - 12:00p
- If needed, we can start at 11:30a or so
- I will probably just have example problems; my slides have most of the major items you need
- Things to do during winter break: anything but economics

Application: Social Security

- A natural extension of the basic model is to look at social security systems
- To start, consider general lump sum taxes when young and old: T_{1t} and T_{2t+1}
  - we will use a negative tax to indicate a transfer (e.g. $T_{2t+1} = -(1+n)T_{1t}$)
- Everything else is standard ...
  - power utility over consumption; preferences are time separable with discount factor β
  - agents work in the first period earning w_t, and can save a_t which commands the interest rate r_{t+1}
  - no productivity growth; population grows at a rate n
  - production is Cobb-Douglas

Solving ...

First Period: $c_{1t} + a_t = w_t - T_{1t}$

Second Period: $c_{2t+1} = (1 + r_{t+1})a_t - T_{2t+1}$

IBC: $c_{1t} + \frac{c_{2t+1}}{1 + r_{t+1}} = w_t - T_{1t} - \frac{T_{2t+1}}{1 + r_{t+1}}$

$$\mathcal{L} = \frac{c_{1t}^{1-\theta}}{1-\theta} + \beta \frac{c_{2t-1}^{1-\theta}}{1-\theta} + \lambda \left[w_t - T_{1t} - \frac{T_{2t+1}}{1 + r_{t+1}} - c_{1t} - \frac{c_{2t+1}}{1 + r_{t+1}} \right]$$

$$\frac{\partial \mathcal{L}}{\partial c_{1t}} = 0 : \qquad \lambda = c_{1t}^{-\theta}$$

$$\frac{\partial \mathcal{L}}{\partial c_{2t+1}} = 0 : \qquad \lambda = \beta(1 + r_{t+1})c_{2t+1}^{-\theta}$$

$$\implies \qquad c_{2t+1} = \left[\beta(1 + r_{t+1})\right]^{\frac{1}{\theta}} c_{1t}$$

Plug the EE into the IBC to get c_{1t}.

$$c_{1t} = \frac{1}{1 + \beta^{\frac{1}{\theta}} (1 + r_{t+1})^{\frac{1-\theta}{\theta}}} \left[w_t - T_{1t} - \frac{T_{2t+1}}{1 + r_{t+1}} \right]$$

Plug c_{1t} into the EE to get c_{2t+1}.

$$c_{2t+1} = \frac{\beta^{\frac{1}{\theta}} (1 + r_{t+1})^{\frac{1}{\theta}}}{1 + \beta^{\frac{1}{\theta}} (1 + r_{t+1})^{\frac{1-\theta}{\theta}}} \left[w_t - T_{1t} - \frac{T_{2t+1}}{1 + r_{t+1}} \right]$$

Plug c_{1t} into the first period budget constraint to get a_t.

$$a_t = \frac{1}{1 + \beta^{\frac{1}{\theta}} (1 + r_{t+1})^{\frac{1-\theta}{\theta}}} \left[\beta^{\frac{1}{\theta}} (1 + r_{t+1})^{\frac{1-\theta}{\theta}} (w_t - T_{1t}) + \frac{T_{2t+1}}{1 + r_{t+1}} \right]$$

- Before continuing forward, there is something interesting that we can see with a_t
  - any tax in the first period of life crowds out investment (there is less with which to save)
  - any tax in the second period of life induces more savings (I have to save more for tomorrow)
  - reverse results if considering a transfer in each period ("negative" tax)
- Now, as is customary, let's consider the most standard social security program . . .
  - the tax on the young is the same each period ($T_{1t} = T_{1t+1}$)
  - tax receipts each period are evenly disbursed to the old (each old person gets $(1+n)T_{1t}$)
  - that is, $T_{2t+1} = -(1+n)T_{1t}$
- Recall, importantly, that there are $1+n$ young people per old person because of population growth

Plugging this in ...

$$c_{1t} = \frac{1}{1+\beta^{\frac{1}{\theta}}(1+r_{t+1})^{\frac{1-\theta}{\theta}}}\left[w_t - T_{1t} + \frac{(1+n)T_{1t}}{1+r_{t+1}}\right]$$

$$c_{2t+1} = \frac{\beta^{\frac{1}{\theta}}(1+r_{t+1})^{\frac{1}{\theta}}}{1+\beta^{\frac{1}{\theta}}(1+r_{t+1})^{\frac{1-\theta}{\theta}}}\left[w_t - T_{1t} + \frac{(1+n)T_{1t}}{1+r_{t+1}}\right]$$

$$a_t = \frac{1}{1+\beta^{\frac{1}{\theta}}(1+r_{t+1})^{\frac{1-\theta}{\theta}}}\left[\beta^{\frac{1}{\theta}}(1+r_{t+1})^{\frac{1-\theta}{\theta}}(w_t - T_{1t}) - \frac{(1+n)T_{1t}}{1+r_{t+1}}\right]$$

Without having to simplify, we can see some pretty interesting results.

- First, social security will always crowd out investment
- Second, social security can actually increase consumption (in both periods!) iff $n > r_{t+1}$ (i.e. if the economy is *dynamically inefficient*)

Dynamic Efficiency: $r_{t+1} > n$

- Dynamic inefficiency in this model, as in others, essentially means that welfare *could* be higher and that savings (capital) is too high
- It can be shown to imply that the interest rate has to be larger than n; the derivation involves showing what is required for Pareto Optimality
- Importantly, dynamic *inefficiency* doesn't mean agents are not optimizing
- Rather, model parameters are such that an individual's optimal behavior does not align with the optimal possible outcome
- In such a scenario, it is possible for the government to step in and rectify the inefficiency using a social security transfer, to give an example

Unexpected One-time Transfer

- Consider another fiscal policy using the OG framework: there are no taxes but the old vote for a one time transfer from the young
- Assume that this happens in period $t+1$; that is the young pay T_{1t+1} and the old receive $(1+n)T_{1t+1}$
- Assuming log utility, the original (no transfer) solution to this problem is

$$a_t = \frac{\beta}{1+\beta} w_t \qquad c_{1t} = \frac{1}{1+\beta} w_t \qquad c_{2t+1} = \frac{\beta(1+r_{t+1})}{1+\beta} w_t$$

- Now, let's look at the effect on the old, the young, and on the aggregate economy ...

Effect on the Old

- Because the old didn't know they could do this, this doesn't affect their consumption when they were young: c_{1t} and a_t are as before

- They do have more income in $t+1$ because of the transfer, and this is all consumed because they are exiting the model

$$c_{2t+1} = \frac{\beta(1+r_{t+1})}{1+\beta} w_t + (1+n) T_{1t+1}$$

- Thus, for the old, they are unambiguously made better off (this makes sense; they are just getting extra consumption)

Effect on the Young

- Unlike with the old, the effect on the young is a little more nuanced (because they can adjust their other behavior)

$$\begin{aligned}
&\text{First Period:} && c_{1t+1} + a_{t+1} = w_{t+1} - T_{1t+1} \\
&\text{Second Period:} && c_{2t+2} = (1 + r_{t+2})a_{t+1} \\
&\text{IBC:} && c_{1t+1} + \frac{c_{2t+2}}{1 + r_{t+2}} = w_{t+1} - T_{1t+1}
\end{aligned}$$

$$a_t = \frac{\beta}{1+\beta}[w_{t+1} - T_{1t+1}]$$

$$c_{1t+1} = \frac{1}{1+\beta}[w_{t+1} - T_{1t+1}] \qquad c_{2t+2} = \frac{\beta(1 + r_{t+2})}{1+\beta}[w_{t+1} - T_{1t+1}]$$

- This makes the young unambiguously worse off, they consume less in both periods

Aggregate Economy: Capital

- Using the expression for assets in $t+1$ (which are lower because of the transfer), we can look at the dynamics of capital in this economy
- Assuming we start out in the steady state, at $t+1$ the capital stock is given by

$$K_{t+2} = L_{t+1} a_{t+1} \qquad \Longrightarrow \qquad k_{t+2} = \frac{\beta}{1+\beta} \frac{w_{t+1} - T_{1t+1}}{1+n} < k_{t+1}$$

- That is, there is an immediate and discrete drop in k after the transfer
- Side Note: the lower capital will increase the interest rate, leading to more savings; this will also lead to lower wages
- In the LR, the economy will converge back to the original steady state

Expected (Pre-announced) Transfer

- The same setup as before, the only difference is that the old can plan ahead
- I.e. the current old people voted for their kids to give them money when they were young
- The problem for the young is still the same as in the unexpected transfer, but the old can now plan ahead . . .

First Period: $c_{1t} + a_t = w_t$

Second Period: $c_{2t+1} = (1 + r_{t+1})a_t + (1 + n)T_{1t+1}$

IBC: $c_{1t} + \frac{c_{2t+1}}{1 + r_{t+1}} = w_t + \frac{(1 + n)T_{1t+1}}{1 + r_{t+1}}$

$$a_t = \frac{\beta}{1+\beta} w_t - \frac{1+n}{(1+\beta)(1+r_{t+1})} T_{1t+1}$$
$$c_{1t} = \frac{1}{1+\beta} w_t + \frac{1+n}{(1+\beta)(1+r_{t+1})} T_{1t+1}$$
$$c_{2t+1} = \frac{\beta(1+r_{t+1})}{1+\beta} w_t + \frac{\beta(1+n)}{1+\beta} T_{1t+1}$$

- Consumption for the old is not quite as high in the second period as with the unexpected transfer (they smooth across both periods)
- They accomplish this by saving less when young

Government Debt

- All of this transfer "stuff" can be thought of as examples of a more general model with government debt
- We haven't seen any D_t, though, because we have assumed that the government
  - transfers from the young to the old in a given period
  - transfers tax receipts from the young back to them when they are old, doing so by investing in capital
- In these cases, there isn't any actual government spending, just "moving things around;" this means that the market clearing condition is unchanged

$$K_{t+1} = L_t a_t$$

- We might think of a scenario where we let the government have debt and to engage in spending somehow

- How does this change the market clearing condition? Now savings are spent on capital and "government bonds"

$$K_{t+1} + D_{t+1} = L_t a_t$$

$$\text{where} \qquad \underbrace{D_{t+1}}_{\text{debt tomorrow}} = \underbrace{(1 + r_{t+1})D_t}_{\text{interest on debt today}} + G_t - \underbrace{[L_t T_{1t} + L_{t-1} T_{2t}]}_{\text{tax receipts in a period}}$$

- Before, we assumed (or imposed) that $D_t = 0 \ \forall t$; government spending then simplifies to

$$G_t = L_t T_{1t} + L_{t-1} T_{2t}$$

- From here, we simply said that this government spending was transferred somehow

- We can allow for positive levels of debt, though

- What else do we need to specify? $\Longrightarrow$ a rule for gov't spending
  - e.g. assume it's some fixed G every period
  - e.g. assume it's some general G_t that varies over time
  - e.g. assume gov't spends a constant fraction of young's income: $\gamma W_t L_t$

- Simply plug in whatever the problem gives you and you have all you need to solve any problem:
  - the market clearing condition with government debt
  - an equation that determines what the government debt is (this added variable requires another condition / equation to pin it down)

- Three models:
    1. Solow – exogenous savings rate, capital determines growth, economy converges to a s.s. in efficiency units
    2. Ramsey – savings is endogenous, decisions made in continuous time
    3. Diamond – savings endogenous, often times we make prices endogenous too, time is discrete
- The Final (I think) is 3 questions; I don't know if it's one question per model
- Know the main (basic) models *very* well; once you have that down make sure to look at different applications / extensions of the models
- Something you need to keep in mind: math is not a substitute for economic intuition, and vice versa (err on the side of intuition)

Econ 204B

- We will move into "Dynamic Programming" next quarter
- We will look at even more complicated models and different ways of writing / solving them
- For instance, we will learn how to write an economic problem *sequentially* (similar to what we've seen in 204A) and *recursively*
- There will be some coding (it's not all coding), and I will likely be emphasizing Python (though you can use whatever language you like)
- Recommendations: Python, R, Matlab