Econ 204B: Section 8

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Worker Turnover

Last time we assumed that the employment state was absorbing. Now, let's generalize the model slightly to allow employed workers to be exogenously separated from their job at a rate s. The expression for the flow value of unemployment remains unchanged:

$$rU = b + \alpha \int_{\underline{w}}^{\overline{w}} \max\left\{V(w) - U, 0\right\} dF(w).$$

Let's derive the flow value of employment (at a wage w). A good habit to have is to start with discrete time and take a limit as $dt \to 0$.

$$V(w) = wdt + \frac{1}{1 + rdt}\Big[(1 - sdt)V(w) + sdtU\Big]$$

$$(1 + rdt)V(w) = (1 + rdt)wdt + (1 - sdt)V(w) + sdtU$$

$$rdtV(w) = (1 + rdt)wdt + sdt\Big[U - V(w)\Big]$$

$$V(w) = \frac{(1 + rdt)w}{r} + \frac{s}{r}\Big[U - V(w)\Big]$$

Take the limit as $dt \to 0$.

$$V(w) = \frac{w}{r} + \frac{s}{r}\Big[U - V(w)\Big]$$

$$rV(w) = w + s\Big[U - V(w)\Big] \qquad \Longrightarrow \qquad V(w) = \frac{w + sU}{r + s}$$

First let's look at, and rewrite, rU.

$$
\begin{aligned}
rU &= b + \alpha \int_{\underline{w}}^{\overline{w}} \max\{V(w) - U, 0\}\, dF(w) \\
&= b + \alpha \int_{\underline{w}}^{\overline{w}} \max\Big\{ \underbrace{\frac{w + sU}{r+s}}_{V(w)} - \underbrace{\frac{w_R + sU}{r+s}}_{U}, 0 \Big\} dF(w) \\
&= b + \frac{\alpha}{r+s} \int_{w_R}^{\overline{w}} (w - w_R) dF(w) \\
&= b + \frac{\alpha}{r+s} \int_{w_R}^{\overline{w}} [1 - F(w)] dw && \text{(int. by parts)}
\end{aligned}
$$

Now we have all we need to determine the reservation wage. Recall this is defined by the wage that solves $rV(w_R) = rU$.

$$r\frac{w_R + sU}{r+s} = rU \implies w_R + sU = rU + sU \implies w_R = rU$$

Thus

$$w_R = b + \frac{\alpha}{r+s}\int_{w_R}^{\overline{w}} [1 - F(w)]dw.$$

From here we can do the same comparative static exercises I showed you last time.

Equilibrium Unemployment

- If employment is absorbing, all agents will eventually become employed
  - no real notion of an equilibrium unemployment / employment rate
- Now that we have added features allowing workers to re-enter unemployment, we can begin discussing equilibrium unemployment
- Here, let's think about equating the flows of workers into and out of unemployment
- Remembering that we typically normalize the mass of agents to be equal to 1, we have

$$\underbrace{\alpha[1 - F(w_R)]u}_{\text{mass leaving UE}} = \underbrace{s(1-u)}_{\text{mass entering UE}} \qquad \Longrightarrow \qquad u = \frac{s}{\alpha[1 - F(w_R)] + s}$$

On-the-Job Search

- Something that is in the data, that so far we cannot account for, are job-to-job transitions
- Many individuals will change jobs without ever entering into unemployment
- Here, we will allow *already employed* workers to continue searching for jobs
- Suppose that unemployed workers receive job offers at a rate α_0, while employed workers receive job offers at a rate α_1
- Maintain all previous assumptions / features

As before, the flow value of unemployment will look similar.

$$rU = b + \alpha_0 \int_{w_R}^{\overline{w}} [V(w) - U] dF(w)$$

Now for the flow value of employment.

$$\begin{aligned} V(w) = wdt + \frac{1}{1+rdt} \Bigg[& \alpha_1 dt(1 - sdt) \int_{\underline{w}}^{\overline{w}} \max\left\{V(\widetilde{w}), V(w)\right\} dF(\widetilde{w}) \\ & + (1 - \alpha_1 dt)(1 - sdt) V(w) \\ & + \alpha_1 sdt^2 \int_{\underline{w}}^{\overline{w}} \max\left\{V(\widetilde{w}), U\right\} dF(\widetilde{w}) \\ & + (1 - \alpha_1 dt) sdtU \Bigg] \end{aligned}$$

Now let's simplify. Pay careful attention to combining like-terms and cancellations.

$$
\begin{aligned}
(1 + rdt)V(w) = (1 + rdt)wdt &+ (\alpha_1 dt - \alpha_1 sdt^2) \int_{\underline{w}}^{\overline{w}} \max \{V(\widetilde{w}), V(w)\} \, dF(\widetilde{w}) \\
&+ (1 - sdt - \alpha_1 dt + \alpha_1 sdt^2) V(w) \\
&+ \alpha_1 sdt^2 \int_{\underline{w}}^{\overline{w}} \max \{V(\widetilde{w}), U\} \, dF(\widetilde{w}) \\
&+ (sdt - \alpha_1 sdt^2) U
\end{aligned}
$$

$$
\begin{aligned}
rdtV(w) = (1 + rdt)wdt + (\alpha_1 dt - \alpha_1 sdt^2) \int_{\underline{w}}^{\overline{w}} \max\left\{V(\widetilde{w}) - V(w), 0\right\} dF(\widetilde{w}) \\
+ sdt\Big[U - V(w)\Big] \\
+ \alpha_1 sdt^2 \int_{\underline{w}}^{\overline{w}} \max\left\{V(\widetilde{w}) - U, 0\right\} dF(\widetilde{w})
\end{aligned}
$$

Now divide both sides by rdt.

$$
\begin{aligned}
V(w) = \frac{(1 + rdt)w}{r} + \left(\frac{\alpha_1 - \alpha_1 sdt}{r}\right) \int_{\underline{w}}^{\overline{w}} \max\left\{V(\widetilde{w}) - V(w), 0\right\} dF(\widetilde{w}) \\
+ \frac{s}{r}\Big[U - V(w)\Big] \\
+ \frac{\alpha_1 sdt}{r} \int_{\underline{w}}^{\overline{w}} \max\left\{V(\widetilde{w}) - U, 0\right\} dF(\widetilde{w})
\end{aligned}
$$

Take the limit as $dt \to 0$.

$$rV(w) = w + \alpha_1 \int_{\underline{w}}^{\overline{w}} \max\{V(\widetilde{w}) - V(w), 0\}\, dF(\widetilde{w}) + s\Big[U - V(w)\Big]$$

- That is, the flow value of employment is given by the instantaneous payoff w plus the expected increase in payoff of a higher wage opportunity plus the expected decrease in payoff associated with losing one's job

- Now, what about the reservation wage?
  - for already employed workers, they will accept a job if $\widetilde{w} > w$ (so the "reservation wage" for an employed worker is her current wage)
  - for unemployed workers, we want to look for the w_R such that $rV(w_R) = rU$

Reservation Wage

Here, unlike with the very basic model, the reservation wage derivation is *slightly* more involved. I recommend following these steps.

1. find $rV(w_R)$
2. construct $rV(w_R) = rU$ and then solve for w_R
3. determine $V'(w)$ and then plug in

1. find $rV(w_R)$

$$rV(w_R) = w_R + \alpha_1 \int_{\underline{w}}^{\overline{w}} \max\left\{V(\widetilde{w}) - V(w_R), 0\right\} dF(\widetilde{w}) + s\Big[\underbrace{U - V(w_R)}_{=0}\Big]$$

$$= w_R + \alpha_1 \int_{w_R}^{\overline{w}} [V(\widetilde{w}) - V(w_R)] dF(\widetilde{w})$$

Now, we can apply integration by parts (similar to how we've done it before), as follows.

$$\int_{w_R}^{\overline{w}} [V(\widetilde{w}) - V(w_R)] dF(\widetilde{w}) \qquad \Longrightarrow \qquad \begin{array}{ll} u = V(\widetilde{w}) - V(w_R) & v = F(\widetilde{w}) \\ du = V'(\widetilde{w}) d\widetilde{w} & dv = dF(\widetilde{w}) \end{array}$$

$$
\begin{aligned}
\int_{w_R}^{\overline{w}} [V(\widetilde{w}) - V(w_R)]dF(\widetilde{w}) &= [V(\widetilde{w}) - V(w_R)]F(\widetilde{w})\Big|_{w_R}^{\overline{w}} - \int_{w_R}^{\overline{w}} V'(\widetilde{w})F(\widetilde{w})d\widetilde{w} \\
&= V(\overline{w}) - V(w_R) - \int_{w_R}^{\overline{w}} V'(\widetilde{w})F(\widetilde{w})d\widetilde{w} \\
&= \int_{w_R}^{\overline{w}} V'(\widetilde{w})d\widetilde{w} - \int_{w_R}^{\overline{w}} V'(\widetilde{w})F(\widetilde{w})d\widetilde{w} \\
&= \int_{w_R}^{\overline{w}} V'(\widetilde{w})[1 - F(\widetilde{w})]d\widetilde{w}
\end{aligned}
$$

We can plug this into the expression for $rV(w_R)$ that we had before...

$$rV(w_R) = w_R + \alpha_1 \int_{w_R}^{\overline{w}} V'(\widetilde{w})[1 - F(\widetilde{w})]d\widetilde{w}$$

2. construct $rV(w_R) = rU$ and then solve for w_R

Before beginning, notice that we can transform the integral in the expression for rU just like we did above noting that $U = V(w_R)$.

$$w_R + \alpha_1 \int_{w_R}^{\overline{w}} V'(\widetilde{w})[1 - F(\widetilde{w})]d\widetilde{w} = b + \alpha_0 \int_{w_R}^{\overline{w}} [V(\widetilde{w}) - U]dF(\widetilde{w})$$

$$= b + \alpha_0 \int_{w_R}^{\overline{w}} V'(\widetilde{w})[1 - F(\widetilde{w})]d\widetilde{w}$$

$$w_R = b + (\alpha_0 - \alpha_1) \int_{w_R}^{\overline{w}} V'(\widetilde{w})[1 - F(\widetilde{w})]d\widetilde{w}$$

3. determine $V'(w)$ and then plug in

$$\text{Recall:} \qquad rV(w) = w + \alpha_1 \int_{\underline{w}}^{\overline{w}} [V(\widetilde{w}) - V(w)] dF(\widetilde{w}) + s\Big[U - V(w)\Big]$$

$$\text{Define:} \qquad \varphi(w) \equiv \int_{w}^{\overline{w}} [V(\widetilde{w}) - V(w)] dF(\widetilde{w}) \qquad \text{(the surplus function)}$$

$$rV'(w) = 1 + \alpha_1 \varphi'(w) - sV'(w)$$

Now let's work on the surplus function...

$$\varphi(w) = \int_{w}^{\overline{w}} [V(\widetilde{w}) - V(w)] dF(\widetilde{w})$$

$$= \int_{w}^{\overline{w}} V'(\widetilde{w})[1 - F(\widetilde{w})] d\widetilde{w} \qquad \text{(int. by parts)}$$

$$\varphi'(w) = -V'(w)[1 - F(w)]$$

Plugging this result into the prior expression, we can determine that

$$rV'(w) = 1 - \alpha_1 V'(w)[1 - F(w)] - sV'(w)$$

$$V'(w) = \frac{1}{r + s + \alpha_1[1 - F(w)]}$$

Plug this in and we've found the reservation wage.

$$w_R = b + (\alpha_0 - \alpha_1) \int_{w_R}^{\overline{w}} \frac{[1 - F(w)]}{r + s + \alpha_1[1 - F(w)]} dw$$

As we did with the first model today, we can very easily determine the equilibrium unemployment rate.

$$\underbrace{\alpha_0[1 - F(w_R)]u}_{\text{mass leaving UE}} = \underbrace{s(1-u)}_{\text{mass entering UE}} \implies u = \frac{s}{\alpha_0[1 - F(w_R)] + s}$$