Problem Set 3

Problem 1. Consider a model economy with continuum of individuals where each individual faces idiosyncratic labor productivity shocks. Each individual solves the following program

$$\max_{c,a'} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$c_t + a_{t+1} = r \cdot a_t + wexp(e)$$

where c is consumption, a is asset holdings, r is the rate of return on assets, w is the wage rate per efficiency units, e is labor productivity. Individual labor productivity is governed by an AR(1)

$$e_{t+1} = \rho e_t + \epsilon_{t+1} \qquad \epsilon_t \sim N(o, \sigma^2)$$

The utility function can be parameterized as

$$u(c_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma}$$

Using the Tauchen-Hussey algorithm we get the following two-stage discrete Markov approximation of the estimated AR(1) process for individual productivity

$$\begin{bmatrix} e_h \\ e_l \end{bmatrix} = \begin{bmatrix} 0.1 \\ \ln(2 - exp(e_h)) \end{bmatrix}$$

and

$$\Pi = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}$$

Here, assume that $\gamma = 2$, $\beta = 0.98$, and $(r, w) = (1.0204, 1.9521)$. Set the asset grid to span from 0 to 200.

(a) Compute the asset distribution for this economy using numerical methods of your choice. (Recommendations: For the individuals problem, piecewise linear splines for approximation and golden-section search for optimization. Integration is trivial when the stochastic process can be approximated by a discrete Markov process. Use Monte Carlo Methods to compute the distribution functions for assets and consumption implied by the individuals' optimal decision rules.

See code provided.

Problem 2. At the beginning of each period an unemployed worker draws one offer to work forever at wage w (which they may accept or reject). Wages are i.i.d. draws from the c.d.f. F, where $F(\underline{w}) = 0$ and $F(\overline{w}) = 1$. The worker seeks to maximize $\mathbb{E} \sum_{t=0}^{\infty} \beta^t y_t$, where y_t is the worker's wage or unemployment compensation, if any. The worker is entitled to unemployment compensation in the amount $\gamma > 0$ only during the *first* period that they are unemployed. After one period on unemployment compensation, the worker receives none.

(a) Write the Bellman equations for this problem.
Note the timing implied by the question:

$$V_E(w) = w + \beta V_E(w)$$

$$V_U^1(w) = \max\left\{V_E(w), \gamma + \beta \int_{\underline{w}}^{\overline{w}} V_U^+(w')dF(w')\right\}$$

$$V_U^+(w) = \max\left\{V_E(w), \beta \int_{\underline{w}}^{\overline{w}} V_U^+(w')dF(w')\right\}$$

(b) Show and explain how the worker's reservation wage and their "hazard of leaving unemployment" (i.e. the probability of accepting a job offer) varies with the duration of unemployment.
As usual, the reservation wage is given by the wage such that an individual is indifferent between working and remaining unemployed. Here, depending on how many periods the worker has been unemployed, the reservation wage will be different. Denote these w_R^1 and w_R^+. Further, we can rewrite $V_E(w) = \frac{w}{1-\beta}$.

$$w_R^1 = (1-\beta)\gamma + (\beta - \beta^2) \int_{\underline{w}}^{\overline{w}} V_U^+(w')dF(w')$$

$$w_R^+ = (\beta - \beta^2) \int_{\underline{w}}^{\overline{w}} V_U^+(w')dF(w')$$

We can see that $w_R^1 - w_R^+ = (1-\beta)\gamma > 0$. That is, the reservation wage is decreasing as an unemployment spell continues. In the first equation above, the reservation wage is proportional to unemployment benefits *plus* the option value of waiting for another offer (where one knows that there will be no benefits in the future).

Next, note that the hazard of leaving unemployment is given by the probability that a wage is drawn at least as large as the reservation wage.

$$\Pr(w \geq w_R^i) = 1 - F(w_R^i) \qquad i \in \{1, +\}$$

Noting the discussion above about the result that $w_R^1 > w_R^+$, we can easily determine that $\Pr(w \geq w_R^1) \leq \Pr(w \geq w_R^+)$, that is the hazard of leaving unemployment is increasing with the length of an unemployment spell.

Now assume that the worker is also entitled to unemployment compensation if they quit their job. As before, the worker receives unemployment compensation in the amount of γ during the first period of an unemployment spell, and zero during the remaining part of the spell. (In order to re-qualify for the benefits, the worker must find a job and work at least one period.)

The timing of events is as follows. At the very beginning of a period, a worker who was employed in the previous period must decide whether or not to quit. If they quit, she draws a new wage offer as described previously, and if she accepts the offer she immediately starts earning that wage without suffering any period of unemployment.

(c) Write the Bellman equations for this problem. [Hint: Let $V_E(w)$ denote the value of a worker who was employed the previous period with wage w, before any decision to quit (and receive some new draw w') occurs.

$$V_E(w) = \max\left\{ w + \beta V_E(w), \int_{\underline{w}}^{\overline{w}} V_U^1(w')dF(w') \right\}$$

$$V_U^1(w) = \max\left\{ w + \beta V_E(w), \gamma + \beta \int_{\underline{w}}^{\overline{w}} V_U^+(w')dF(w') \right\}$$

$$V_U^+(w) = \max\left\{ w + \beta V_E(w), \beta \int_{\underline{w}}^{\overline{w}} V_U^+(w')dF(w') \right\}$$

(d) Characterize (i.e. give an expression for it) the reservation strategy of an employed worker and then prove that the reservation wage for someone unemployed longer than one period is equal to 0 (if you cannot prove it, give some intuition as to why it must be the case).

First denote the reservation wage of an employed person as w_R^E (this gives the cutoff for when a worker will decide to quit or stay at a job). It is defined by the following.

$$w_R^E + \beta V_E(w_R^E) = \int_{\underline{w}}^{\overline{w}} V_U^1(w')dF(w')$$

Because the problem is the same every period for an employed worker, her quit / stay decision will always be the same. We can utilize this fact to simplify the above.

$$w_R^E = (1-\beta) \int_{\underline{w}}^{\overline{w}} V_U^1(w')dF(w')$$

Regarding the proof, suppose that $w_R^+ > 0$. Then we must have that

$$\underbrace{w_R^+ + \beta V_E^+(w_R^+) = \beta \int_{\underline{w}}^{\overline{w}} V_U^+(w')dF(w')}_{\text{reservation statement for "+" unemployed}} < \underbrace{\int_{\underline{w}}^{\overline{w}} V_U^1(w')dF(w') = w_R^E + \beta V_E(w_R^E)}_{\text{reservation statement for employed workers}}.$$

Since $w+\beta V_E(w)$ is (weakly) increasing in w, the above statement necessarily implies that $w_R^+ < w_R^E$. Because we are talking about reservation strategies, if we were to plug in w_R^+ into the employed worker's problem, we know she will quit.

$$V_E(w_R^+) = \int_{\underline{w}}^{\overline{w}} V_U^1(w')dF(w')$$

Finally, we can utilize the above result to plug into the reservation statement for "+" workers (and do some moving around).

$$w_R^+ + \beta \overbrace{\int_{\underline{w}}^{\overline{w}} V_U^1(w')dF(w')}^{V_E(w_R^+)} = \beta \int_{\underline{w}}^{\overline{w}} V_U^+(w')dF(w')$$

$$w_R^+ = \beta \left[\int_{\underline{w}}^{\overline{w}} V_U^+(w')dF(w') - \int_{\underline{w}}^{\overline{w}} V_U^1(w')dF(w') \right] < 0 \qquad \text{(contradiction)}$$

The above is a contradiction, and so we cannot have $w_R^+ > 0$. Because wages are (weakly) positive, we must have $w_R^+ = 0$. Intuitively, since after the first period of unemployment a worker does not receive any benefits, accepting an offer and then quitting is at least as good as rejecting an offer and drawing again next period.

Problem 3. Suppose you are a policymaker trying to choose between increasing unemployment insurance and implementing wage subsidies. You recall that the equation for the reservation wage in the discrete search model is:

$$w_r = b + \frac{\beta}{1-\beta} \int_{w_r}^{\bar{w}} [1 - F(w)] dw$$

and you want to use this to inform your decision. Using numerical methods of your choosing:

(a) What does an increase of unemployment insurance (b) from 2 to 5 do to the reservation wage? Use an exponential wage distribution with $\lambda = 2$, $\beta = .95$, and $\bar{w} = 100$.

(b) Now, using the same distribution, solve for the reservation wage under a wage subsidy scheme. For the purposes of this assignment assume that the subsidy results in a discrete shift of the wage distribution by 3.

(c) Compare your findings from (a) and (b). What should the policymaker support?
Hint: Use Monte-Carlo Integration and the Bisection method.

See provided Code