ECON 204C

June, 2016

Spring 2016 Final

1) Effort on the job. Consider a standard search model in discrete time. Suppose the probability of a layoff, λ, is not exogenous as we assumed in class but is a function of how much effort you spend at your job; so that $\lambda = \lambda(e)$, with $\lambda' < 0$. In particular, the value of employment at wage w is

$$V(w) = \max_e \{u(w,e) + \beta[\lambda(e)U + [1-\lambda(e)]V(w)]\}, \tag{1}$$

with $u_1 > 0$ and $u_2 < 0$.

a. Derive the reservation wage equation and the first order condition for the choice of effort as a function of the wage.

Answer:

$$U = u(b,0) + \beta \int_{\underline{w}}^{\bar{w}} \max\{V(w), U\} dF(w) \tag{2}$$

$$(1-\beta)U = u(b,0) + \beta \int_{WR}^{\bar{w}} (V(w) - U) dF(w) \tag{3}$$

$$(1-\beta)U = u(b,0) + \beta \int_{WR}^{\bar{w}} (V(w) - U) dF(w) \tag{4}$$

$$V(WR) = u(WR, e^*) + \beta[\lambda(e^*)U + [1-\lambda(e^*)]V(WR)] \tag{5}$$

by definition $V(WR) = U$:

$$\Rightarrow V(WR) = u(WR, e^*) + \beta[V(WR)] \tag{6}$$

$$\Rightarrow V(WR) = \frac{u(WR, e^*)}{1-\beta} \tag{7}$$

$$\Rightarrow (1-\beta)\frac{u(WR, e^*)}{1-\beta} = u(b,0) + \beta \int_{WR}^{\bar{w}} (V(w) - U) dF(w) \tag{8}$$

$$\Rightarrow u(WR, e^*) = u(b,0) + \beta \int_{WR}^{\bar{w}} (V(w) - U) dF(w) \tag{9}$$

$$\Rightarrow u(WR, e^*) = u(b,0) + \beta \int_{WR}^{\bar{w}} (V(w)) dF(w) - [1 - F(WR)]V(WR) \tag{10}$$

Performing integration by parts:

$$\Rightarrow u(WR,e^*) = u(b,0)+[V(\bar{w})-\cancel{V(WR)F(WR)}]\beta\int_{WR}^{\bar{w}} V'(w)F(w)dw-[1-\cancel{F(WR)}]V(WR) \tag{11}$$

$$\Rightarrow u(WR,e^*) = u(b,0) + \beta\int_{WR}^{\bar{w}} V'(w)[1-F(w)]dw \tag{12}$$

What is $V'(w)$? Assume $e = e^*$:

$$V'(w) = u_1(w,e) + \beta[1-\lambda(e)]V'(w) \tag{13}$$

$$\Rightarrow [1-\beta[1-\lambda(e)]]V'(w) = u_1(w,e) + \beta[1-\lambda(e^*)]V'(w) \tag{14}$$

$$\Rightarrow V'(w) = \frac{u_1(w,e^*)}{[1-\beta[1-\lambda(e^*)]]} \tag{15}$$

Then the reservation wage is implicitly implied by the following:

$$\Rightarrow u(WR,e^*) = u(b,0) + \frac{\beta}{[1-\beta[1-\lambda(e^*)]]}\int_{WR}^{\bar{w}} u_1(w,e^*)[1-F(w)]dw \tag{16}$$

Then, the FOC for effort is given by the following:

$$\frac{\partial V(w)}{\partial e} = u_2(w,e(w)) + \beta[\lambda'(e)U - \lambda'(e)V(w) + (1-\lambda(e))\frac{\partial V(w)}{\partial e}] \tag{17}$$

$$[1-\beta(1-\lambda(e))]\frac{\partial V(w)}{\partial e} = u_2(w,e(w)) + \beta[\lambda'(e)U - \lambda'(e)V(w)] \tag{18}$$

$$\frac{\partial V(w)}{\partial e} = \frac{u_2(w,e(w)) + \beta[\lambda'(e)U - \lambda'(e)V(w)]}{[1-\beta(1-\lambda(e))]} \tag{19}$$

$$0 = u_2(w,e(w)) - \beta[\lambda'(e)(V(w)-U)] \tag{20}$$

Thus, we have a FOC that defines effort:

$$\lambda'(e) = \frac{u_2(w,e(w))}{\beta(V(w)-U)} \tag{21}$$

b. Find $e'(w)$ in the case where $u_{12} = 0$.

Taking the derivative of the FOC for e, we get the following:

$$\frac{\partial\lambda'(e)}{\partial w}e'(w) = \cancel{\frac{u_{12}(w,e(w))}{\beta(V(w)-U)}} + \frac{u_{22}(w,e(w)))e'(w)}{\beta(V(w)-U)} - \frac{u_2(w,e(w))}{(\beta(V(w)-U))^2}V'(w) \tag{22}$$

$$\frac{\partial \lambda'(e)}{\partial w}e'(w) = \frac{u_{22}(w, e(w)))e'(w)\beta(V(w) - U)}{(\beta(V(w) - U))^2} - \frac{u_2(w, e(w))}{(\beta(V(w) - U))^2}V'(w) \quad (23)$$

$$\frac{\partial \lambda'(e)}{\partial w}e'(w) = \frac{u_{22}(w, e(w)))e'(w)\beta(V(w) - U) - u_2(w, e(w))V'(w)}{(\beta(V(w) - U))^2} \quad (24)$$

$$e'(w) = \frac{u_{22}(w, e(w)))e'(w)\beta(V(w) - U) - u_2(w, e(w))V'(w)}{\frac{\partial \lambda'(e)}{\partial w}(\beta(V(w) - U))^2} \quad (25)$$

$$e'(w)[1 - \frac{u_{22}(w, e(w)))\beta(V(w) - U)}{\frac{\partial \lambda'(e)}{\partial w}(\beta(V(w) - U))^2}] = \frac{-u_2(w, e(w))V'(w)}{\frac{\partial \lambda'(e)}{\partial w}(\beta(V(w) - U))^2} \quad (26)$$

$$\Rightarrow e'(w) = \frac{-u_2(w, e(w))V'(w)}{\frac{\partial \lambda'(e)}{\partial w}(\beta(V(w) - U))^2}[1 - \frac{u_{22}(w, e(w)))\beta(V(w) - U)}{\frac{\partial \lambda'(e)}{\partial w}(\beta(V(w) - U))^2}]^{-1} \quad (27)$$

2) Indivisible Labor. Consider a neo-classical growth model with a labor-leisure trade-off. Specifically, agents can choose $h = \{0, \bar{h}\}$ hours to work each period. Standard conditions apply and the planner solves the following problem:

$$\max E(\sum_{t=0}^{\infty} \beta^t (ln(C_t) + AL_t)) \quad (28)$$

subject to

$$C_t + K_{t+1} = z_t F(K_t, H_t) + (1 - \delta)K_t \quad (29)$$

$$H_t = 1 - L_t \quad (30)$$

$$ln(z_t) = \rho ln(z_t) + \epsilon_t \quad (31)$$

where L_t is a lottery over $l_t = \{1_t, 1 - \bar{h}\}$, where the planner is selecting probability of working, α_t.

a. Starting with power utility, $\frac{(c^\gamma l^{1-\gamma})^{1-\sigma}}{1-\sigma}$, derive the period utility function given in the problem, stating clearly any assumptions that are necessary. Note specifically what A and L_t are equal to.

Answer:

$$\frac{(c^\gamma l^{1-\gamma})^{1-\sigma}}{1 - \sigma} \quad (32)$$

$\sigma \to 0$:

$$\gamma ln(c) + (1 - \gamma) ln(l) \quad (33)$$

divide through by γ:

$$ln(c) + \frac{(1-\gamma)}{\gamma} ln(l) \tag{34}$$

Now, note that the gamble is as follows: pick α such that $prob(h = 0) = (1 - \alpha)$ and $prob(h = \bar{h}) = \alpha$:

$$\alpha(ln(c) + \frac{(1-\gamma)}{\gamma} ln(1-\bar{h})) + (1-\alpha)(ln(c) + \frac{(1-\gamma)}{\gamma} \cancel{ln(1)}) \tag{35}$$

$$ln(c) + \alpha \frac{(1-\gamma)}{\gamma} ln(1-\bar{h})) \tag{36}$$

recall that by definition, $L = \alpha l$, or $\alpha = \frac{L}{l}$. Substituting this in,

$$ln(c) + \frac{(1-\gamma)}{\gamma} ln(1-\bar{h}) \frac{L}{l} \tag{37}$$

We now have a definition for L and A: $L = L$ (silly) and

$$A = \frac{(1-\gamma)}{\gamma} \frac{ln(1-\bar{h})}{1-\bar{h}} \tag{38}$$

b. Write down the representative agents problem, and define the competitive equilibrium.

Answer:

The representative agent chooses a gamble over working and not working. Thus, his problem becomes the following:

$$\max E(\sum_{t=0}^{\infty} \beta^t (ln(C_t) + AL_t)) \tag{39}$$

subject to

$$C_t + K_{t+1} = w_t H_t + (1 + r_t - \delta) K_t \tag{40}$$

$$H_t = 1 - L_t \tag{41}$$

$$ln(z_t) = \rho ln(z_t) + \epsilon_t \tag{42}$$

Then, the competitive equilibrium is defined as a sequence of choices, $\{C_t, K_{t+1}, L_t\}$ by the agent, factor demands $\{H_t, K_t\}$ and a set of prices $\{w_t, r_t$ satisfying

1. The sequence of choices solve the represetative agents problem, defined above, given prices.
2. Factor demands solve the problem of a profit-maximizing firm, given prices.
3. Prices are determined by markets clearing for labor and capital.

You could also define this as a recursive competitive equilibrium, and it would look similar.

c. Describe the problem with introducing indivisible labor into this problem, and why lotteries are able to solve it.

Answer:

Indivisible labor introduces a non-convexity into the consumption set; in general, we cannot solve these problem for general equilibrium allocations. In essence, by allowing lotteries, we are taking the convex hull of the consumption set, which allows for general equilibrium solutions.

3) **Search and Matching.** Consider the standard model of random search in continuous time. Agents receive job offers at rate λ, and draw from a degenerate wage distribution with a single mass point at w. When they are unemployed, they receive benefits $b < w$. Jobs separate at rate δ, and agents discount utility at rate r.

a. Write out the flow Bellman equation of an unemployed and employed worker.

Answer:

Note that a degenerate distribution implies that there is only one wage in the defined economy.

$$rV_U = b + \lambda[V_E(w) - U] \tag{43}$$

$$rV_E(w) = w + \delta[U - V_E(w)] \tag{44}$$

b. Suppose instead that the match rate is endogenous, i.e. that it is a function of the number of vacancies posted by firms. That is, there is a matching function $M(u, v)$, which is constant returns to scale; this means the matching function can be expressed as $m(\theta) = M(1, \frac{v}{u})$, where $\theta = \frac{v}{u}$ is market tightness. The rate at which firms meet workers is now $q(\theta)$, and the rate at which workers meet firms is now $p(\theta)$. Rewrite the problem of the unemployed and employed worker from part (a) to reflect this new information.

Answer:

This actually only changes one component of the problem. Note that this is **NOT** a double coincidence problem.

$$rV_U = b + \lambda[V_E(w) - U] \tag{45}$$

$$rV_E(w) = w + p(\theta)[U - V_E(w)] \tag{46}$$

c. A job in this context is a match between a firm and a single worker. Output from each job is y, and the wage is still w. Firms discount the future at the same rate as workers, r, and face a cost of opening a vacancy, κ. Think of this cost of opening a vacancy as the flow cost of having an unfilled job (the firm's version of an unemployment benefit, but a cost). Write out the flow of expected profits for creating a vacancy for the firm. Also, write out the flow value to the firm of having a filled job. Finally, determine what happens to the flow creation condition when profits are competed away to zero in expectation.

Answer:

The firm has two equations: one that characterizes whether or not they will open vacancies, and one that characterizes the value of having a filled job.

$$rV_V = -\kappa + q(\theta)[V_F(w) - V_v] \tag{47}$$

$$rV_F(w) = y - w + \delta[V_V - V_F(w)] \tag{48}$$

or

$$rV_F(w) = y - w - \delta[V_F(w)] \tag{49}$$

In equilibrium, they are equivalent. The second part of this question asks what happens to the vacancy creation condition when *expected* profits are competed away to zero. What this means is that firms open vacancies until $q(\theta)$ declines to the point at which $rV_V = 0$:

$$0 = -\kappa + q(\theta)[V_F(w) - V_v] \tag{50}$$

$$\Rightarrow \kappa = q(\theta)[V_F(w) - V_v] \tag{51}$$